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FREE CONVECTION OF NON-NEWTONIAN FLUIDS OVER NON-ISOTHERMAL TWO-DIMENSIONAL BODIES

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INTRODUCTION

HEAT transfer in non-Newtonian fluids is of practical importance in many industries, for example in paper making, drilling of petroleum products, slurry transporting, and processing of food and polymer solutions.

Acrivos [1] was apparently the first to investigate in 1960 the natural convection behavior of non-Newtonian fluid flow from a body with an isothermal surface. Since then quite a number of investigations have been done with success [2–12]. An excellent review on the subject of convective heat transfer in non-Newtonian fluids has recently been made by Shenoy and Mashelkar [13].

Most of the studies on free convection in non-Newtonian fluids are concerned with simple bodies such as a flat plate or cylinder with uniform wall temperature or uniform surface heat flux. In a great many technical applications, however, the body shape is neither flat nor cylindrical and its surface is thermally non-uniform, on which attention will be focused here. In considering such problems, it is natural to examine the family of bodies having certain wall-temperature or wall-flux variations which will give rise to similarity thermal characteristics.

The objective of this work is to analyze the free convection heat transfer in power-law non-Newtonian fluids from a two-dimensional (2-D) body of which the surface is subject to power-law variations in (a) temperature and (b) heat flux. In view of the fact that most of the non-Newtonian fluids have large Prandtl numbers, this study is directed towards such fluids. Similar temperature profiles and heat transfer results are presented, also examined in detail are the effects of body shape, flow index, and surface thermal variations.

ANALYSIS

Consider a 2-D body submersed in a quiescent bulk of power-law non-Newtonian fluid at constant temperature T_∞. The boundary-layer equations of free convection in the steady, laminar, and incompressible flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g_x\beta(T-T_\infty)+\frac{K}{\rho}\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right], \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where (x, y) are curvilinear coordinates with x measured from the front stagnation and along the body contour; (u, v) are velocity components in these directions; T is the temperature; ρ, the fluid density; α, the fluid thermal diffusivity; β, the thermal expansion coefficient; and K and n are the fluid consistency and flow index of the power-law fluid, respectively. The x-component of gravitational acceleration, g_x, is related to the body-contour angle, ε, by $g_x = g \sin \varepsilon$, in which ε is the angle between the y- and g-axis and is given by

$$\varepsilon=\sin^{-1}\left[1-\left(\frac{\mathrm{d}R}{\mathrm{d}x}\right)^2\right], \tag{4}$$

where $R(x)$ is the distance from the vertical Z-axis (of which the origin is at the front stagnation point) to the body surface. The appropriate boundary conditions are:

$$y=0:\ u=v=0;\quad T=T_w(x)=T_\infty+\Delta T_0\left(\frac{x}{L}\right)^p,$$

or

$$-k\frac{\partial T}{\partial y}=q_w(x)=q_0\left(\frac{x}{L}\right)^s, \tag{5}$$

$$y\to\infty:\ u=0;\quad T=T_\infty,$$

where L is the length of body, k is the thermal conductivity of the fluid, and p, s, and ΔT_0 are constants.

For most non-Newtonian fluids the Prandtl number is quite large, and thus the inertial effect of the flow, represented by the two terms on the LHS of equation (2), may be neglected [1]. Under this assumption, it can be readily shown that similarity solutions exist for the problem described by equations (1)–(5) in which the body shape of the 2-D body varies according to

$$\sin\varepsilon=(x/L)^m. \tag{6}$$

To study the two cases of surface thermal conditions prescribed by equation (5), we assign $i = 1$ for the case of variable wall-temperature and $i = 2$ for the case of variable wall-heat-flux in the following transformations:

$$\eta_i=c_i y_i x_i^{\lambda_i-1},\quad \psi_i=d_i x_i^{\lambda_i} f(\eta_i),\quad \theta_i=(T-T_\infty)/G_i, \tag{7}$$

where ψ is the stream function and

$$x_i=x/L,\quad y_i=H_i y/L,\quad c_i=\lambda_i^{nt_i},$$

$$d_i=\lambda_i^{-(2n+1)t_i},\quad t_i=1/(3n+i),$$

$$\lambda_1=(m+p+2n+1)t_i,\quad \lambda_2=(m+s+2n+2)t_2,$$

$$G_1=T_w-T_\infty,\quad G_2=c_2H_2k/(q_0L)x_2^{(m-n-3sn-s)t_2},$$

$$H_i=Gr_i^{0.5/(n/i+i)}\,Pr_i^{n/(3n+i)},$$

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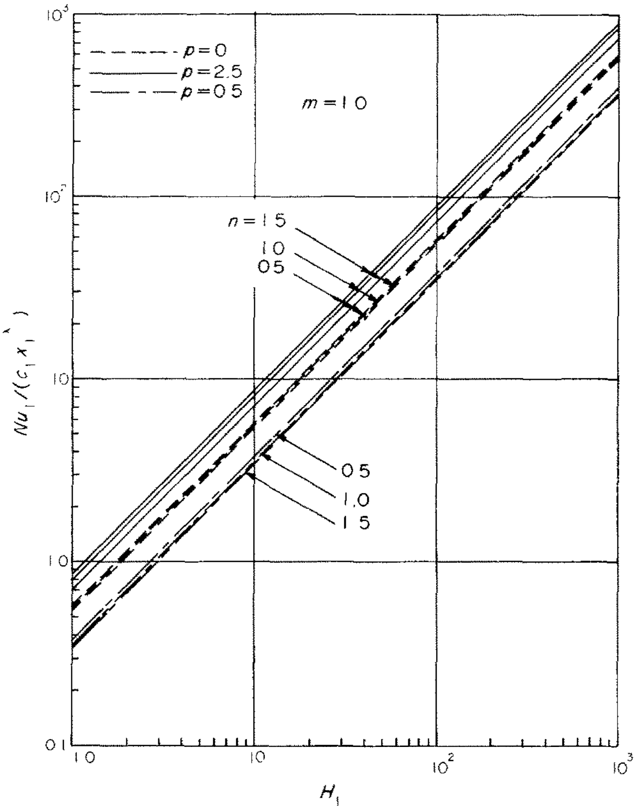

FIG. 1. Local Nusselt number of variable wall-temperature.

in which Gr_i and Pr_i are the respective generalized Grashof number and Prandtl number given by

$$Pr_i = \alpha^{-1}(K/\rho)^{(3i-1)/a}L^{2(n-1)/a}(g\beta D_i)^{1.5i(n-1)/a},$$

$$Gr_i = (\rho/K)^2 L^4 (g\beta D_i)^{2-n},$$

with $a = 3i+n-2$, $D_1 = \Delta T_0/L$, and $D_2 = q_0/k$.

Use of equation (7) in equations (1)–(6) yields, under the assumption of large Prandtl number

$$\frac{d}{d\eta_i}(f'')^n + \theta_i = 0, \tag{8}$$

$$\theta_i'' + f\theta_i' - B_i f'\theta_i = 0, \tag{9}$$

$$f(0) = f'(0) = 0; \quad f''(\infty) = 0, \tag{10}$$

$$\theta_1(0) = 1 \quad \text{or} \quad \theta_2'(0) = -1; \quad \theta_i(\infty) = 0, \tag{11}$$

where

$$B_1 = \frac{p(3n+1)}{m+p+2n+1},$$

and

$$B_2 = \frac{s(3n+1)+n-m}{m+s+2(n+1)}.$$

It is to be noted that the boundary condition $f'(\infty) = 0$ has been replaced by $f''(\infty) = 0$ because of the large Pr approximation involved for which the inertia force, thus $\partial u/\partial y$, becomes asymptotically unimportant as $Pr \to \infty$ [1]. The heat transfer from the surface to the fluid can be characterized by the local Nusselt number defined by

$$Nu_i = \frac{hx}{k} = c_i H_i x_i^{\lambda_i} E_i, \tag{12}$$

where h is the heat transfer coefficient, $E_1 = -\theta'(0)$ and $E_2 = \theta^{-1}(0)$. The similarity equations (8)–(11) can be readily integrated using a standard numerical method. Some highlights of the result are given in the following section.

RESULTS AND DISCUSSION

Variable wall-temperature bodies

It is interesting to note that for the case of an isothermal body ($p = 0$) as seen from equations (8) to (11), the dimensionless temperature profile and its wall gradient are

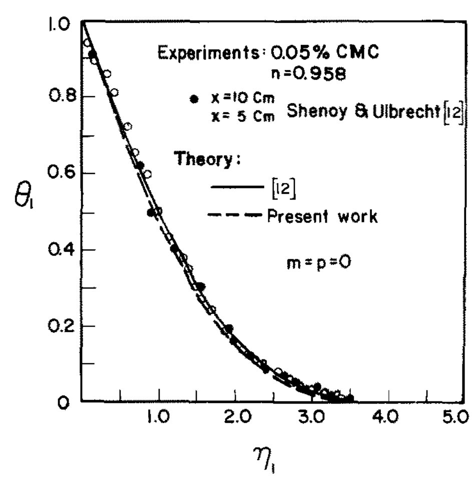

FIG. 2. Comparison of dimensionless temperatures for $m = p = 0$.

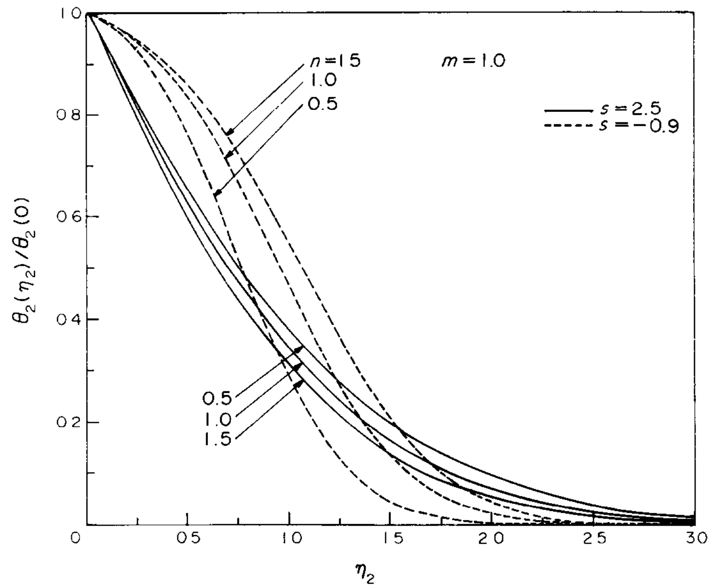

FIG. 3. Temperature profiles for wall-flux bodies.

independent of the body shape parameter m, but the local Nusselt number, given by equation (12), is affected by the body contour. Figure 1 shows the local Nusselt number in the stagnation region of a cylinder ($m = 1.0$), which is typical for other body shapes studied. It increases with increasing p. Its dependence on the flow index n is such that it increases with increasing n where $p \geqslant 0$ decreases when $p < 0$.

Experiments for an isothermal plate ($m = p = 0$) have been reported by Shenoy and Ulbrecht [12] and the comparison of the present results, shown in Fig. 2, with their measured and predicted data gives good agreement. It is noted in passing that, to check the accuracy of the present solution, excellent agreement has been obtained as compared to those reported by Acrivos [1] for the cases of an isothermal flat plate ($p = m = 0$) and of stagnation flow of an isothermal cylinder ($p = 0$ and $m = 1.0$).

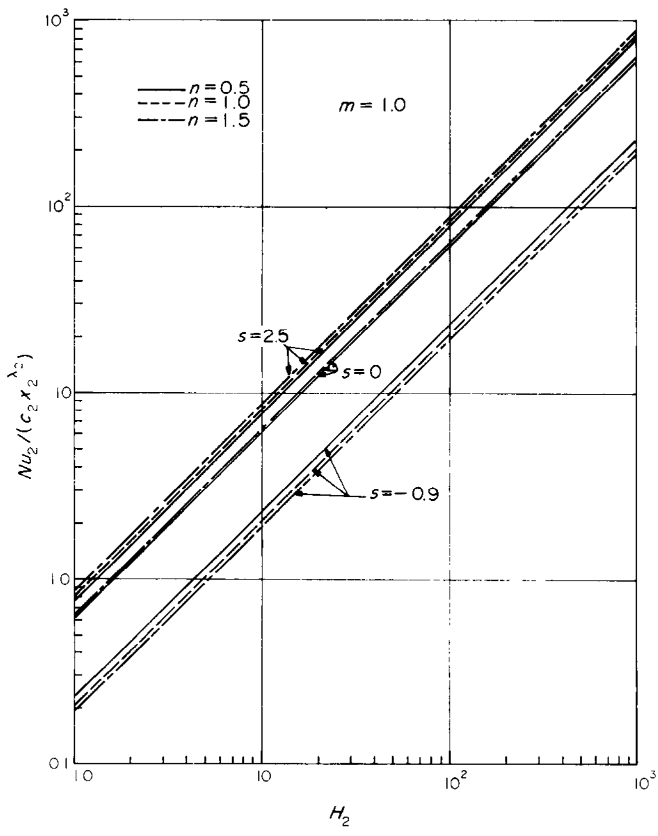

FIG. 4. Local Nusselt number for $m = 1.0$.

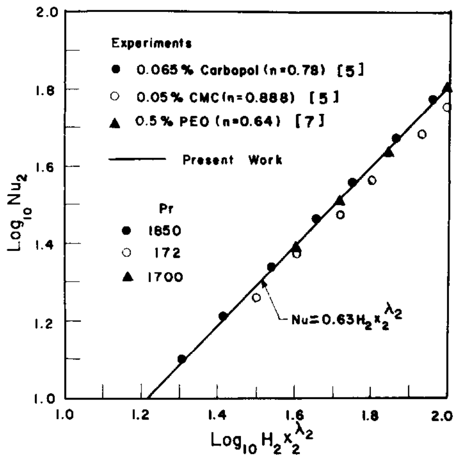

FIG. 5. Comparison with experimental results for $m = 0$.

Variable wall-flux bodies

To assess the effects of varying flow indices and wall heat flux upon the temperature distribution, the dimensionless temperature profiles are presented for $m = 1.0$ in Fig. 3. Similar to the case of wall-temperature bodies, S-shaped curves are observed for negative values of s, while others are regular for positive values of s. Decreasing n is seen to result in a thinner thermal boundary layer for $s = -0.9$, while it tends to thicken it for $s = 2.5$. The relative orientation of the curves for $m = 1.0$ is typical for all other values of m studied.

Typical curves showing the moderate effect on the local Nusselt number for the case of a cylinder ($m = 1.0$) are shown in Fig. 4. For the case of a uniform wall-flux body ($s = 0$), the dependence on n is seen to be very small. This suggests that, as has been demonstrated in refs. [5–7], the heat transfer data for this case may be well correlated for practical applications by a single constant C in the generalized Newtonian correlation of the form $\overline{Nu} = CH_2$. As seen from Fig. 4, however, the effect of n becomes more important as the wall thermal condition deviates from uniform heat flux. Thus, the value of C is no longer a constant but depends upon the flow index n.

Finally, the present solution is compared with experiments reported by Dale and Emery [5] and by Fujii *et al.* [7] for several pseudoplastic fluids over a flat plate with uniform wall heat flux. Figure 5 shows the comparison. The solid line represents the correlation for the case of $m = s = 0$ deduced from the present theoretical results. For those two fluids with larger Pr, the agreement is excellent. Due to the assumption of asymptotically large Pr involved in the present analysis, the error of the present results is within 6% as compared to the experimental data for the 0.05% CMC fluid with $Pr = 172$.

CONCLUDING REMARKS

In many engineering problems, the surface thermal condition is non-uniform. The familiar superposition of solutions for different wall thermal variations for the energy equation is invalid in such cases because that for free convection problems the momentum and energy equations are coupled. On the other hand, the body shape in many instances is neither a flat plate nor a circular cylinder. Thus, the solutions obtained by a straightforward numerical integration of the similarity equations given by equations (8)–(11) for variable wall-temperature and wall-heat-flux 2-D bodies may prove to be useful for practical applications.

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